The logic of Classes as Many:

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The Logic of Classes as Many

1. Classes as Many as Extensions of Names

In our previous lecture we formalized a logic of names that is an important part of the theory of reference in conceptual realism. The category of names, it will be remembered, includes both proper and common, and complex and simple, names, all of which occur as parts of quantifiers phrases. Quantifier phrases, of course, are what stand for the referential concepts of conceptual realism. We explained in that lecture how Leśniewski's logic of names, which Leśniewski called ontology, can be interpreted and reduced to our conceptualist logic of names, and how in that reduction we can explain and account for the oddities of Leśniewski's logic.

We concluded our previous lecture with observations about the "nominalization," or transformation, of names as parts of quantifier phrases into object terms.

What a "nominalized" name denotes as an object term, we said, is the extension of that name, i.e., of the concept that the name stands for in its role as part of a quantifier phrase.

Now the extension of a name is a **class as many**, i.e., a class as a plurality that is literally made up of its members. We listed three of the central features of classes as many as originally described by Bertrand Russell in his 1903 *Principles of Mathematics*.

These are, first, that a vacuous name — that is, a name that names nothing — has no extension, which is not the same as having an empty class as its extension. In other words, there is no empty class as many. Secondly, the extension of a name that names just one thing is none other than that one thing; that is, a class as many that has just one member is identical with that one member. In other words it is because a class as many is literally made up of its members that it is nothing if it has no members, and why it is identical with its one member if it has just one member.

Finally, that is also why a class as many that has more than one member is merely a plurality, or plural object, which is to say that as a plurality it is not a "single object," and therefore it cannot itself be a member of any class as many.

We begin where left off in our last lecture, namely, with the logic of classes as many as an extension of the logic of names. We assume all of the axioms and theorems of the logic of names. That logic consisted essentially of a free first-order logic of identity extended to include the category of names as parts of quantifiers, and where the quantifiers $\forall$ and $\exists$ can be indexed by name variables as well as object variables.

Now because names can be transformed into object terms we need a variable-binding operator that generates complex names the way that the λ-operator generates complex predicates. We will use the cap-notation with brackets, $[\hat{x}/...x...]$, for this purpose.

Accordingly, where A is a name, proper or common, complex or simple, we take $[\hat{x}A]$ to be **a complex name**, but one in which the variable x is bound.

Thus, where A is a name and φ is a formula, $[\hat{x}A]$, $[\hat{x}A/\varphi]$, and $[\hat{x}/\varphi]$ are names in which all of the free occurrences of x in A and φ are bound.

We read these expressions as follows:

$[\hat{x}A]$ is read as 'the class (or group) of A',

$[\hat{x}A/\varphi]$ is read as 'the class (or group) of A that are φ', and

$[\hat{x}/\varphi]$ is read as 'the class (or group) of *things* (*objects*) that are φ'.

A formal language L is now understood as a set of predicate and name constants. There will be object constants in a formal language as well, but they will be generated from the name constants by a "nominalizing" transformation. In our more comprehensive framework, which we are not concerned with here, object constants are also generated from predicate constants by the nominalizing transformation described in a previous lecture.

2. Logical Grammar

We will now extend the simultaneous inductive definition of names and formulas given in our previous lecture to include names of this complex forms:

If L is a formal language, then:

(1) every name variable or name constant in L is a *name of* L;

(2) if a, b are either object variables, name variables or name constants in L, or names of L of the form $[\hat{x}B]$, where x is an object variable and B is a name (complex or simple) of L, then $(a = b)$ is a *formula* of L; and

if φ, ψ are formulas of L, B is a name (complex or simple) of L, and x and C are an object and a name variable, respectively, then

(3) $\neg\varphi$,
(4) $(\varphi \rightarrow \psi)$,
(5) $(\forall x)\varphi$,
(6) $(\forall xB)\varphi$, and
(7) $(\forall C)\varphi$ are *formulas of* L, and
(8) B/φ,
(9) $/\varphi$, and
(10) $[\hat{x}B]$ are *names* of L.

Note that by definition we now have formulas of the form $(\forall y[\hat{x}A])\varphi$, as well as those of the form $(\forall xA)\varphi$ and $(\forall yA(y/x))\varphi$ as in our previous lecture.

We can reduce the form $(\forall y[\hat{x}A])\varphi$ to the form $(\forall y A(y/x))\varphi$ by adding the following axiom schema. (The numbering goes back to axioms in our previous lecture that, for brevity, were not listed.)

Axiom 12: $(\forall y[\hat{x}A])\varphi \leftrightarrow (\forall y A(y/x))\varphi$, where y does not occur in A.

We might note, incidentally, that Axiom 12 is a conversion principle for complex names as parts of quantifier phrases. It is the analogue for complex names of the form $[\hat{x}/A]$ of λ-conversion for complex predicates of the form $[\lambda x\varphi]$.

This means we also have the following as a theorem (where y is free for x in A):

T8: $\vdash (\exists y[\hat{x}A])\varphi \leftrightarrow (\exists y A(y/x))\varphi$.

Two other axioms about the occurrence of names as object terms are:

Axiom 13: $(\exists A)(A = [\hat{x}B])$, where B is a name and A is a name variable that does not occur in B;

Axiom 14: $A = [\hat{x}A]$, where A is a simple name, i.e., a name variable or constant.

Axiom 13 is a comprehension principle for complex names, and as such is the analogue for complex names of the comprehension principle (CP^*_λ) for complex predicates. What it says is that every complex name of the form $[\hat{x}B]$ is a value of the bound name variables, and therefore stands for a name concept. Axiom 14 tells us that the name concept $[\hat{x}A]$ is none other than the name concept A.

We turn now to definitions of some of the concepts that are important in the logic of classes. Note that although we adopt the same symbols that are used in set theory to express membership, inclusion and proper inclusion, it should be kept in mind that the present notion of class is not that of set theory.

Definition 1 $x \in y \leftrightarrow (\exists A)[y = A \wedge (\exists z A)(x = z)]$.

Definition 2 $x \subseteq y \leftrightarrow (\forall z)[z \in x \rightarrow z \in y]$.

Definition 3 $x \subset y \leftrightarrow x \subseteq y \wedge y \nsubseteq x$.

Note also that the argument for **Russell's paradox** for classes does not lead to a contradiction within this system as described so far, nor will it do so with the axioms yet to be listed.

Rather, what it shows is that the Russell class as many does not "exist" in the sense of being the value of a bound object variable, which is not to say that *the name concept* of the Russell class does not have its own conceptual mode of being as a value of the bound name variables.

Indeed, as the following definition indicates, the name concept of the Russell class can be defined in purely logical terms.

Definition 4 $Rus = [\hat{x}/(\exists A)(x = A \wedge x \notin A)]$.

The fact that the Russell class does not "exist" in the logic of classes as many is stated in the following theorem.

T10: $\vdash \neg(\exists x)(x = Rus)$.

What Russell's argument shows is that not every name concept has an extension that can be "object"-ified in the sense of being the value of a bound object variable.

Now the question arises as to whether or not we can specify a necessary and sufficient condition for when a name concept has an extension that can be "object"-ified, i.e., for when the extension of the concept can be proven to "exist" as the value of a bound object variable.

3. Individuals as Atoms

Unlike the situation in set theory, a condition for when a name concept has an extension can be given for classes as many. An important part of this condition is Nelson Goodman's notion of an "**atom**," which, although it was intended for a strictly nominalistic framework, we can utilize for our purposes and define as follows.

Definition 5 $Atom = [\hat{x}/\neg(\exists y)(y \subset x)]$.

This notion of an atom has nothing to do with physical atoms, of course. Rather, it corresponds in our present system approximately to the notion of an urelement in set theory.

We say "approximately" because in our system atoms are identical with their singletons, and hence each atom will be a member of itself.

This means that not only are ordinary physical objects atoms in this sense, but so are the propositions and intensional objects denoted by nominalized sentences and predicates in the fuller system of conceptual realism.

Of course, the original meaning of 'atom' in ancient Greek philosophy was that of **being indivisible**, which is exactly what was meant by '**individual**' in medieval Latin. An atom, or individual, in other words, is a "single" object, which is apropos in that objects in our ontology are either single or plural. We will henceforth use 'atom' and 'individual' in just this sense.

The following axiom (where y does not occur in A) specifies when and only when a name concept A has an extension that can be "object"-ified as a value of the bound object variables.

Axiom 15: $(\exists y)(y = [\hat{x}A]) \leftrightarrow$
$(\exists xA)(x = x) \wedge (\forall xA)(\exists zAtom)(x = z)$.

Axiom 15 says that the extension of a name concept A can be "object"-ified (as a value of the bound object variables) if, and only if, something is an A and every A is an atom.

An immediate consequence of this axiom, and of T8 and T1, is the following theorem schema, which stipulates exactly when an arbitrary condition φx has an extension that can be "object"-ified.

T11: $\vdash (\exists y)(y = [\hat{x}/\varphi x]) \leftrightarrow$
$(\exists x)\varphi x \wedge (\forall x/\varphi x)(\exists zAtom)(x = z)$.

Note that where φx is the impossible condition $(x \neq x)$, it follows from T11 that **there can be no empty class**, which, as already noted, is our first basic feature of the notion of a class as many.

We define the empty-class concept as follows and note that its extension, by T11, cannot "exist" (as a value of the bound object variables), as well as that no *object* can belong to it.

Definition 6 $\Lambda = [\hat{x}/(x \neq x)]$.

T12a: $\vdash \neg(\exists x)(x = \Lambda)$.
T12b: $\vdash \neg(\exists x)(x \in \Lambda)$.

Finally, our last axiom concerns the second basic feature of classes as many; namely, that every atom, or individual, is identical with its singleton.

In terms of a name concept A, the axiom stipulates that if at most one thing is an A and that whatever is an A is an atom, then whatever is an A is identical to the extension of A, which in that case is a singleton if in fact anything is an A. Where y does not occur in A, the axiom is as follows.

Axiom 16: $(\forall xA)(\forall yA)(x = y) \wedge$
$(\forall xA)(\exists zAtom)(x = z) \rightarrow (\forall yA)(y = [\hat{x}A])$.

A more explicit statement of the thesis that an atom is identical with its singleton is given in the following theorem.

T13: $\vdash (\exists zAtom)(x = z) \rightarrow x = [\hat{y}/(y = x)]$.

By T13, it follows that every atom is identical with the extension of some name concept, e.g., the concept of being that atom.

T14: $\vdash (\exists zAtom)(x = z) \rightarrow (\exists A)(x = A)$.

Of course, non-atoms, i.e., plural objects, are the extensions of name concepts as well (by the definitions of *Atom*, $\subset$, and $\in$), and hence anything whatsoever is the extension of a name concept.

T15: $\vdash \neg(\exists z Atom)(x = z) \to (\exists A)(x = A)$.
T16: $\vdash (\exists A)(x = A)$.

A consequence of T13 is the thesis that every atom is a member of itself. A similar consequence is that every object is a member of its singleton.

T17: $\vdash (\forall x Atom)(x \in x)$.
T18: $\vdash (\forall x)(x \in [\hat{z}/(z = x)])$.

Finally, we note that by definition of membership an object x belongs to the extension of a name concept A if, and only if, x is an A.

T19: $\vdash x \in A \leftrightarrow (\exists y A)(x = y)$.

From this it follows that only atoms can belong to an "object"-ified class as many, and hence that classes as many that are not atoms are not themselves members of any (real) classes as many, which is our third basic feature of classes as many.

T20a: $\vdash (\forall x)[z \in x \to (\exists w Atom)(z = w)]$.
T20b: $\vdash \neg(\exists w Atom)(z = w) \to \neg(\exists x)(z \in x)$.

4. Extensional Identity

The "nominalist's dictum" is that "no two distinct things can have the same atoms." This dictum should apply to classes as many as traditionally understood, regardless whether or not a more comprehensive framework containing such classes is nominalistic or not. In fact, the dictum is provable here if we assume an axiom of extensionality for classes.

Now there is a problem with the axiom of extensionality. In particular, if the full unrestricted version of Leibniz's law is not modified, then having an axiom of extensionality would commit us to a strictly extensional framework. Name concepts that have the same extension at a given moment in a given possible world would then, by Leibniz's law, be necessarily equivalent, and therefore have the same extension at all times in every possible world, which is counter-intuitive.

The extension of a common-name concept such as 'country that is democratic' can change its extension over time, for example. Certainly, common name concepts of animals, e.g., 'buffalo', have different extensions over time. Some, as in the case of names of plants and animals that have become extinct have changed their extensions radically from having millions of members to now having none.

The idea that common name concepts cannot have different extensions over time, no less in different possible worlds, is a consequence we cannot accept in our broader framework of conceptual realism.

On the other hand, classes as many are extensional objects, and an axiom of extensionality that applies at least to classes as many is the only natural assumption. All objects, in other words, whether they are single or plural, are classes as many, and the idea that classes as many are not identical when they have the same members is difficult to reconcile with such an ontology.

Fortunately, there is an alternative, namely, that the full version of Leibniz's law as it applies to all contexts is to be **restricted to atoms**, i.e., individuals in the ontological sense. The unrestricted version for extensional contexts can then still be applied to pluralities, i.e., classes as many that have more than one member. Thus, in addition to the axiom of extensionality, we will take the following as an new axiom schema of our general framework.

Axiom 17: $(\exists z Atom)(x = z) \wedge (\exists z Atom)(y = z) \rightarrow$
$[x = y \rightarrow (\varphi \leftrightarrow \psi)]$,

where ψ is obtained from φ by replacing one or more free occurrences of x by free occurrences of y.

This distinction between how Leibniz's law applies to atoms and how it applies to classes as many in general is an ontological feature of our logic in that it distinguishes the individuality of atoms from the plurality of groups.

We now include **the axiom of extensionality**, which we will refer to hereafter as (ext), among the axioms.

Axiom 18 (ext): $(\forall z)[z \in x \leftrightarrow z \in y] \rightarrow x = y$

Goodman's nominalistic dictum that things are identical if they have the same atoms is now provable as the following theorem.

T21: $\vdash (\forall x)(\forall y)[(\forall z Atom)(z \in x \leftrightarrow z \in y) \rightarrow$
$x = y]$.

Note that by T13 and the definition of $\in$, whatever belongs to an atom is identical with that atom, and therefore atoms are identical if, and only if, they the have the same members.

T22: $\vdash (\forall x Atom)[y \in x \rightarrow y = x]$.

T23: $\vdash (\forall x Atom)(\forall y Atom)[x = y \leftrightarrow$
$(\forall z)(z \in x \leftrightarrow z \in y)]$.

Note also that by T21 (and other theorems) it follows that everything "real", whether it is an atom or not, has an atom in it.

T24: $\vdash (\forall x)(\exists z Atom)(z \in x)$.

Another useful theorem is the following, which, together with T21, shows that every non-atom must have at least two atoms as members. Of course, conversely, any "real" class (as many) that has at least two members cannot be an atom, because then each of those members is properly contained in that class.

T25: $\vdash (\forall x)(\forall y)(y \subset x \rightarrow$
$(\exists z Atom)[z \in x \wedge z \notin y])$.

T26: $\vdash (\forall x)[\neg(\exists y Atom)(x = y) \leftrightarrow$
$(\exists z_1/z_1 \in x)(\exists z_2/z_2 \in x)(z_1 \neq z_2)]$.

Two consequences of the extensionality axiom, (ext), are the strict identity of a class with the class of it members and the rewrite of bound variables for class expressions.

T27a: $\vdash x = [\hat{z}/(z \in x)]$.

T27b: $\vdash [\hat{x}A] = [\hat{y}A(y/x)]$, where y does not occur in A.

5. The Universal Class

We have seen that, unlike the situation in set theory, the empty class as many does not "exist" (as a value of the bound object variables). But what about the universal class?

In ZF**,** Zermelo-Fränkel set theory, there is no universal set, but in Quine's set theory NF (New Foundations) and the related set theory, NFU (New Foundations with Urelements), there is a universal set.

In our present theory, the situation is more complicated. For example, if nothing exists, then of course the universal class does not exist. But, in addition, because something exists only if an atom does, i.e., by T24 and ($\exists$/UI),

T28: $\vdash (\exists x)(x = x) \rightarrow (\exists x Atom)(x = x)$,

it follows that the universal class does not exist if there are no atoms, i.e., individuals — which is unlike the situation in set theory where classes exist whether or not there are any urelements, i.e., individuals.

As it turns out, we can also show that the universal class does not exist if there are at least two atoms. If there is just one atom, however, the situation is more problematic.

First, let us define the universal class in the usual way, i.e., as the extension of the common name 'thing that is self-identical', and then note that whether or not the name concept *thing-that-is-self-identical*, i.e., $[\hat{x}/(x = x)]$, can be "object"-ified (as a value of the bound object variables), nevertheless, everything "real" (in the sense of being the value of a bound object variable) is in it.

Definition 7 $\mathbf{V} = [\hat{x}/(x = x)]$.

T29: $\vdash (\forall x)(x \in \mathbf{V})$.

Note: all that T29 really says is that *everything* is a *thing that is self-identical*.

Now, by definition of $\in$, nothing can belong to the empty class, i.e., $x \notin \Lambda$, and therefore, by Leibniz's law, if anything at all exists, the universal class is not the empty class.

T30: $\vdash (\exists x)(x = x) \rightarrow \mathbf{V} \neq \Lambda$.

But it does not follow that the universal class "exists" if anything does. Indeed, as already noted, we can show that if there are at least two atoms, then the universal class does not exist. First, let us note that if something exists (and hence, by T28, there is an atom), then the class of atoms exists, i.e., then the name concept *Atom* can be "object"-ified as a value of the bound object variables.

T31: $\vdash (\exists x)(x = x) \rightarrow (\exists y)(y = Atom)$.

But note also that if there are at least two atoms, then the class of atoms is not itself an atom.

T32: $\vdash (\exists x Atom)(\exists y Atom)(x \neq y) \rightarrow \neg(\exists z Atom)(z = Atom)$.

By T32, we can show that if there are at least two atoms, then the universal class does not "exist" (as a value of the object variables).

T33: $\vdash (\exists x Atom)(\exists y Atom)(x \neq y) \rightarrow \neg(\exists x)(x = \mathbf{V})$.

Finally, let us consider the question of whether or not the universal class exists if the universe consists of just one atom.

Note that if the universe consists of just one atom, then, where A is a name of that one atom, the conjunction

$$(\exists z Atom)(z = A) \wedge (\forall z Atom)(z = A)$$

would be true, and therefore the one atom A would be extensionally identical with the class of atoms, i.e., then, by T31, T21, and (ext), $(A = Atom)$ would be true as well. Now, by T29 and T19,

$$(\forall z Atom)[z \in Atom \leftrightarrow z \in \mathbf{V}],$$

is provable, which, by T21 — which says that classes are identical if they have the same atoms as members — might suggest that $(Atom = \mathbf{V})$ and hence $(A = \mathbf{V})$ are true as well.

But in order for T21 to apply in this case we need to know that $\mathbf{V}$ "exists," i.e., that $(\exists x)(x = \mathbf{V})$ is true. So, even if there were just one atom, we still could not conclude that the universal class is extensionally identical with that one atom.

6. Leśniewskian Theses Revisited

As we explained in our previous lecture, Leśniewski's logic of names is reducible to our conceptualist logic of names. On our interpretation, the oddities of Leśniewski's logic are seen to be a result of his representing names, both proper and common, the way **singular terms** are represented in modern logic.

The problem is not the fact that proper and common names together constitute a syntactic category of their own, because that is how names are viewed in our conceptualist logic as well.

But in our conceptualist logic proper and common names function as parts of quantifier phrases, i.e., expressions that stand for referential concepts in the nexus of predication.

But if Leśniewski's logic of names is reducible to our conceptualist logic of names, then might not the oddities that arise in Leśniewski's logic also arise when names as parts of quantifier phrases are "nominalized" and are allowed to occur as object terms in the logic of classes as many the way they occur in Leśniewski's logic of names?

In other words, to what extent, if any, are there any theorems in our logic of classes as many that are counterparts of the theses of Leśniewski's logic that struck us as odd or noteworthy?

Here, by a counterpart we mean a formula that results by replacing the names in a thesis of Leśniewski's logic by the "nominalized", or transformed, names of our logic of classes as many, and also, of course, replacing Leśniewski's epsilon 'ε' by our epsilon '$\in$'.

First, let us consider the validity of principle of existential generalization in Leśniewski's logic, i.e.,

$$\varphi(c/a) \rightarrow (\exists a)\varphi(a).$$

This principle is odd, we noted, when a is a vacuous name such as 'Pegasus', because in that case it follows from the fact that nothing is identical with Pegasus that something is identical with nothing, which is absurd. The counterpart of this thesis in our logic of classes as many is clearly invalid. For example, the empty class as many does not "exist" in our logic, and from that it does not follow that something exists that does not exist. Indeed, it is actually disprovable, as it should be. That is, the formula

$$\neg(\exists x)(x = \Lambda) \rightarrow (\exists y)\neg(\exists x)(x = y),$$

is disprovable in our logic of classes as many.

Another thesis of Leśniewski's logic that is odd is the following:

$$(\exists a)(a \neq a).$$

Now, by (UG) and axiom 8, the negation of this thesis, namely $(\forall x)(x = x)$, is a theorem in our logic of classes as many.

But of course stating the matter this way assumes that identity in Leśniewski's logic means identity *simpliciter*, which it doesn't.

In other words, identity is not a primitive but is defined in Leśniewski's logic. What the thesis $(\exists a)(a \neq a)$ really means on Leśniewski's definition of identity is the following:

$$(\exists a)\neg(a \ \varepsilon \ a).$$

Now the real counterpart of this thesis in our logic of classes as many is:

$$(\exists x)\neg(x \in x).$$

By quantifier negation, what this formula says is that not every object belongs to itself, which because all atoms belong to themselves, means that not every object is an atom.

That is not a theorem of our logic, but it would be true if in fact there were at least two atoms, in which case there would then be a group, i.e., a class as many with more than one member, which, by definition, would not be an atom, and therefore, by T20b, not a member of anything, no less of itself.

Thus, although the counterpart of the above Leśniewskian thesis is not a theorem, nevertheless it is not disprovable, and in fact it is true if there are at least two atoms, i.e., individuals in the ontological sense.

In regard to the Leśniewskian thesis,

$$a \ \varepsilon \ b \rightarrow a \ \varepsilon \ a.$$

we note first that the counterpart of this formula, namely,

$$z \in x \rightarrow z \in z,$$

is refutable if there is at least one plural object, i.e., one "real" object that is not an atom.

This is because every "real" object is a member of the universal class (by T29), even though the universal class itself is not "real" if there are at least two atoms (T33).

In other words, where z is a plural object, e.g., the class as many of citizens of Italy, then even though z is a member of the universal class, i.e., $z \in \mathbf{V}$, nevertheless $z \notin z$. That is, because z is a plural object, it is not an atom, and therefore (by T20b) z is not a member anything. Here, it should be kept in mind that even though $\mathbf{V}$ is not a value of the *bound* object variables, it is nevertheless a substituend of the *free* object variables. Hence, where z is a plural object, the following instance of the above formula,

$$z \in \mathbf{V} \rightarrow z \in z$$

is false.

There is a theorem that is somewhat similar to Leśniewski's thesis, namely,

$$\vdash (\exists x)(z \in x) \rightarrow z \in z.$$

In other words, if z belongs to something "real", i.e., a value of the bound object variables, then z is an atom (by T20a) and therefore z belongs to itself (by T17). This theorem is similar to, but still not the same as, Leśniewski's thesis.

Another theorem that is similar to, but not the same as, a thesis of Leśniewski's logic, is:

$$\vdash (\forall y)(\forall z)[x \in y \wedge y \in z \rightarrow x \in z].$$

This formula is provable because if x belongs to a "real" object y and y belongs to a "real" object z, then both x and y must be atoms (by T20a), in which case, $y = [\hat{w}/w = y]$ (by T13); and hence $x = y$ (because $x \in y$), and therefore $x \in z$ (because $y \in z$). This theorem is similar to the Leśniewskian thesis,

$$a \; \varepsilon \; b \wedge b \; \varepsilon \; c \rightarrow a \; \varepsilon \; c,$$

but, again, the strict free-variable counterpart of this Leśniewskian thesis, namely,

$$x \in y \wedge y \in z \rightarrow x \in z$$

is not provable in our logic, and is in fact refutable if there are at least two atoms.

7. Cantor's Power-set Theorem

Finally, there is the question of whether or not Cantor's theorem applies to classes as many when there are a countably infinite number of atoms, i.e., single objects, in the world. Does it then follow by Cantor's power-set theorem that there are an uncountably infinite number of groups of individuals, i.e., of classes as many of more than one member? And if so, wouldn't that show that there is more to the notion of a group than that of a simple plurality? That is, if out of $\aleph_0$ many individuals we can obtain $2^{\aleph_0}$ many groups of individuals, then doesn't that show that there is something abstract, and not concrete, about groups — that is, that classes as many of two or more objects have an abstract mode of being similar to sets or classes as ones?

Now by a simple inductive argument it can be shown that Cantor's theorem is provable if the number of atoms is finite and greater than one. It is not provable in the logic of classes as many, however, when the class as many of atoms is infinite, which suggest that there is nothing abstract about groups of atoms as plural objects after all. Indeed, as the extensions of at most a potentially infinite number of concepts that proper and common names stand for, it would be surprising if their number were to exceed the number of the concepts whose extensions they are.